

Using Mathematical Modeling and HiDEM to evaluate the rate of Secondary Ice Cliff Failure

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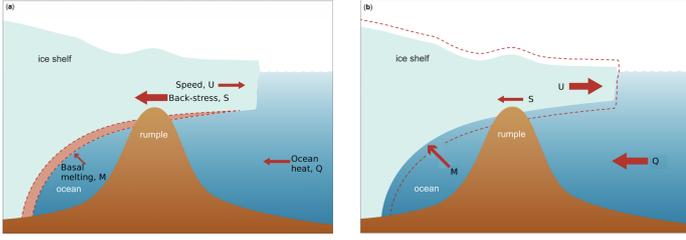


Figure 1: Mechanism of underwater ice erosion. This process reduces cliff strength significantly and is one of the main reasons of ice cliff failure. J. Roberts et al. *Ocean forced variability of Totten Glacier mass loss*. 2017.

Background

Ice cliff instability could accelerate ice loss from Antarctica, and according some model predictions could potentially contribute >1m of global mean sea level rise by 2100. One main direct consequence of ice cliff instability is ice cliff failure, which is due to erosion of melting water from the surface of the glacier. Detailed process are shown in Figure 1.

In their 2021 paper, A. Crawford et al. concluded the positive correlation between ice cliff instability and its height, and further pointed out the secondary ice cliff failure in the primary collapsing region due to the lack of original shear. However, they didn't quantify the secondary retreat. Thus, this project aims to further explore the mechanism of secondary ice cliff retreating.

Methodology

Basic formulas. In his 2002 paper, van der Veen explored the empirical formula of "minimal critical thickness", H_w . When the ice front thickness is lower than this value, ice flow and calving will both accelerate:

$$H_b = H_M - \frac{\rho_w}{\rho_i} H_w,$$

where H_b is the thickness of ice front in excess of that which would be floating, H_w is the water depth at the terminus. In addition, ice sheets calving rate (\dot{c}) is also positively related to rate of glacial stretching ($\dot{\epsilon}_{xx} = \frac{\partial u}{\partial t}$):

$$\dot{c} \propto \dot{\epsilon}_{xx} H_M \frac{Y}{Y + Y_0},$$

where Y_0 is an empirical constant equal to 200 km. In addition, the glacial center of gravity may also be useful when calculating the tension on the secondary ice cliff:

$$\begin{aligned} \Delta M &= \Delta \cdot \int_{\mathcal{A}} \bar{\rho} [S - B] d\mathcal{A} \\ &= \Delta t \cdot \left[\int_{\mathcal{A}} [\dot{b}_s + \dot{b}_e + \dot{b}_b] dS + Q_{in} - \{ \dot{B}_C \text{ or } Q_{out} \} \right], \end{aligned}$$

where \mathcal{A} is the area of interest, $\bar{\rho}$ is the average density, S and B denotes the ice thickness of the surface and the bed (glacier thickness $H = S - B$), Δ indicates a net change, and

Q_{in} denotes the mass entering the zone by flux (on boundaries); \dot{b}'_s denote the mass of melting water from different sources.

Under the assumption that ice thickness is exactly at floating at the grounding line, the vertical average force balance at the grounding line is

$$\bar{\tau}_{xx} (h - d_s - d_b - d_w) = \frac{\rho_i g h^2}{2} \left(\frac{\rho_i}{\rho_w} \right) \theta,$$

where $\bar{\tau}_{xx}$ is the depth-averaged longitudinal stress, h is the ice thickness, d_s and d_b are the depth of dry-surface and basal crevasses respectively, d_w is the additional depth of surface due to hydrofracturing of surface melt or rain-water, and θ is 1 minus the fractional reduction in the hydrostatic imbalance at the grounding line due to back stress on an ice shelf.

Tools and system. HiDEM (The Helsinki Discrete Element Model), an open sourced model for simulating elastic behavior. In this project, HiDEM is build on OSX10.15-Python3.12-x86_64.

Elmer/Ice. It is an open source finite element software for ice sheet, glaciers and ice flow modeling. It can be used to analyze inner tension and stress conditions of a glacier.

All the programs are installed and will be runned on OS X.

Main purpose

This project aims to found a mathematical model to describe the change of ice sheet retreating rate after the primary ice cliff failure. Furthermore, the change of tensile failure conditions after the primary ice cliff failures are also being studied.

Preparations and further readings

HiDEM and Paraview are installed. Test cases are run on the system to test the validity of the workflow. Planend readings including: K.M. Cuffey and W.S.B. Paterson *The Physics of Glaciers (fourth edition)*, D. Pollard, R. DeConto, and R. Alley, *Potential Antarctic Ice Sheet retreat driven by hydrofracturing and ice cliff failure*.

References

- [1] K.M. Cuffey and W.S.B. Paterson (2010) *The Physics of Glaciers (fourth edition)*, Burlington, MA: Elsevier.
- [2] A.J. Crawford et al. *Marine ice-cliff instability modeling shows mixed-mode ice-cliff failure and yields calving rate parameterization*, Nature Communications (2021)12:2701.
- [3] J. Roberts et al. *Ocean forced variability of Totten Glacier mass loss*, Geological Society, London, Special Publications (2017), 461, 175–186.
- [4] D. Pollard, R.M. DeConto, R.B. Alley. *Potential Antarctic Ice Sheet retreat driven by hydrofracturing and ice cliff failure*, Elsevier B.V., open access under the CC BY-NC-ND license.